A Theoretical Study on the Orientation Problem in Linear Wireless Sensor Networks

WASA 2012

by

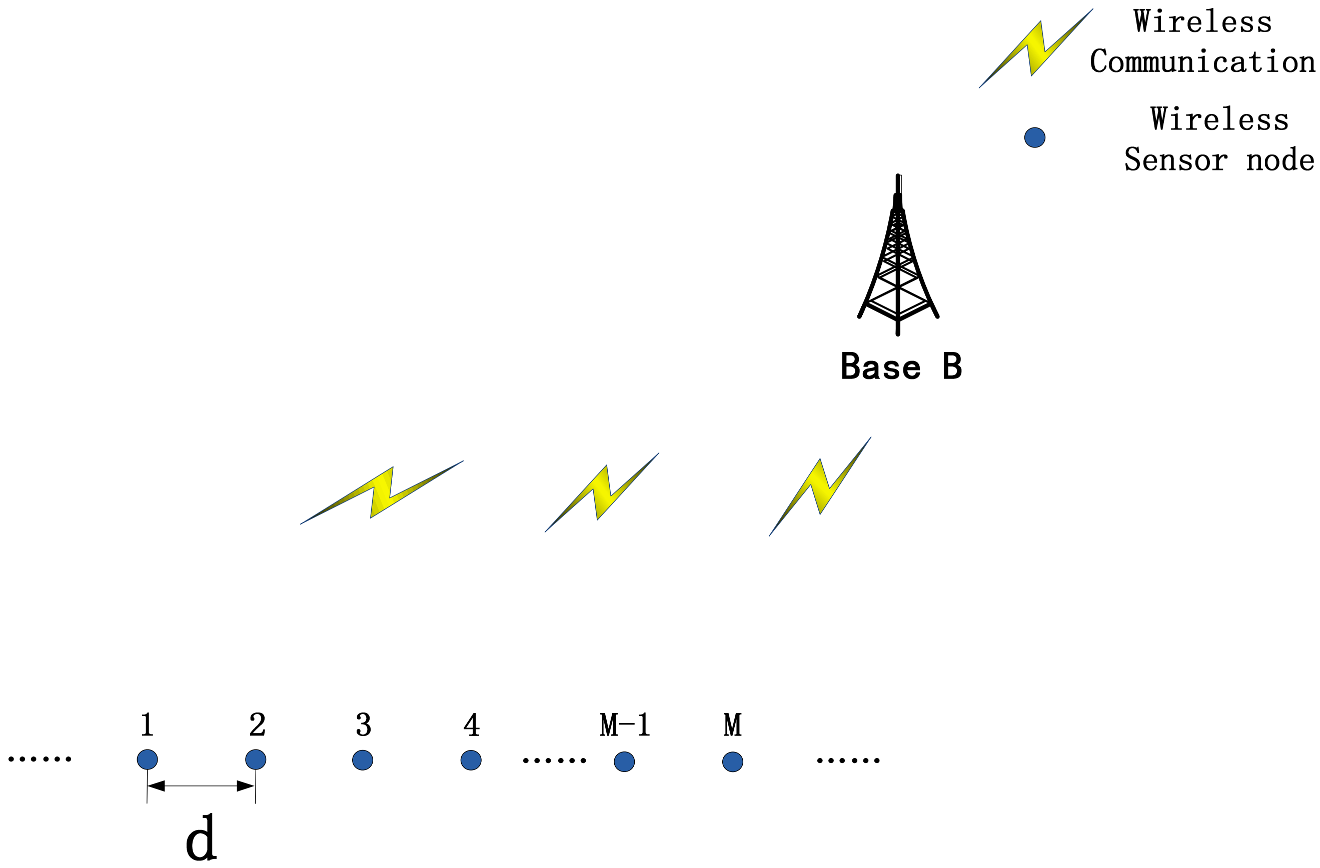
JiangHong Han, Xu Ding, etc.

Hefei University of Technology

What is Linear Sensor Networks

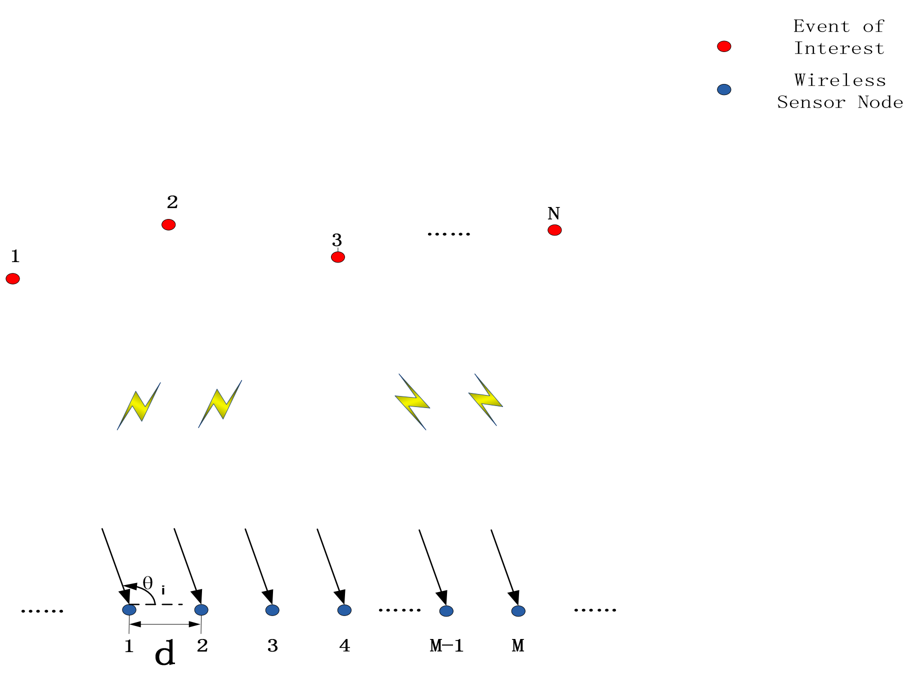
- Linear wireless sensor networks (LWSNs) are a special family of wireless sensor networks with regard to linear network topology. Compared with normal wireless sensor networks, LWSNs exhibit lower complexity. Even though simple in topology, LWSNs possess numerous practical applications, such as monitoring public transportations, oil pipes, factories and plants.

An Example of a LWSN



Model and Preconditions

- The model of a LWSN



Suppose that at a time instance t, there are N events bursting out, and they are sensed by M sensor nodes in the sensing area.

In the left figure, d denotes the distance between any two sensor nodes, and θ_i represents the angel of the i-th received signal with respect to the line of the deployment of M sensor nodes.

Model and Preconditions

- Preconditions

  (1) Assume that all the sensor nodes are equally spaced, and the distance between any two sensor nodes is d;

  (2) Suppose that all the transmitted signals are all narrow-banded with some certain known frequency f;

  (3) All the signals propagate over the distance long enough such that the received signals are parallel to each other.

Motivation

- The Array Manifold Matrix

  According to the previous network model and preconditions, the N received signals can be represented by an array manifold matrix

$$\theta = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \exp(-\frac{j2\pi d}{\lambda}\sin\theta_1) & \exp(-\frac{j2\pi d}{\lambda}\sin\theta_2) & \cdots & \exp(-\frac{j2\pi d}{\lambda}\sin\theta_N) \\ \vdots & \vdots & \vdots & \vdots \\ \exp(-\frac{j(M-1)2\pi d}{\lambda}\sin\theta_1) & \exp(-\frac{j(M-1)2\pi d}{\lambda}\sin\theta_2) & \cdots & \exp(-\frac{j(M-1)2\pi d}{\lambda}\sin\theta_N) \end{bmatrix}_{M\times N}$$

  In the above matrix,

  (1)d represents the distance between sensor nodes;

Motivation

(2) λ =1/f, where f is the frequency of the narrow-banded signals;

(3) θ_i is the arriving angle of the i-th signal with respect to the line of the deployment of the M sensor nodes.

It is also assumed that these received N signals are interfered by some additive noise with zero mean and σ_n^2 Gaussian distribution which is uncorrelated with signals.

Motivation

Therefore, the covariance matrix C of the received signal has the form of

$$C = C_N + GPG^H$$

where

(1) C_N denotes the covariance matrix of M-dimension additive noise;

(2) P denotes the correlation matrix between signals, and here, $P = diag(\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2)$. σ_i^2 denotes the i-th signal power.

Motivation

- Till now, it is clear that the covariance matrix can be parameterized by $\sigma_n^2, \{\sigma_i^2\},$ and $\{\theta_i\}$, i.e.,

$$C = C(\sigma_n^2, \{\sigma_i^2\}, \{\theta_i\})$$

  it is obvious that the covariance matrix conveys knowledge about the arrival angle of the i-th signal in that C is a function of which will help us determine the direction of this signal.

Estimation of the covariance matrix

- The structure of the covariance matrix

  The covariance matrix C is a complex valued toeplitz matrix, and also a Hermitian matrix. However, it is still not easy to estimate this matrix even though it is quite structured. Therefore, we firstly introduce the method which can be used to estimate some structured real valued matrices, and then the result will be extended to the realm of complex valued matrices.

Estimation of the covariance matrix

- The Estimation of real valued covariance matrix

  If all the signals are real valued, we want to estimate the covariance matrix of signals from N samples $x_1, x_2, \ldots, x_N$. Here, we suppose that the N samples are drawn from an M-dimension Gaussian distribution with zero mean and covariance matrix C. Therefore, he probability density function for each x_i is

$$p\left(x_i\right) = (2\pi)^{-\frac{M}{2}} \det\left(C\right)^{-\frac{1}{2}} \exp(-\frac{x_i{}^T C^{-1} x_i}{2})$$

Estimation of the covariance matrix

What is more, If all these samples are independent to each other, the joint probability density function of these N samples is

$$p\left(x_1, x_2, \ldots, x_N\right) = (2\pi)^{-\frac{MN}{2}} \det\left(C\right)^{-\frac{N}{2}} \exp(-\sum_{i=1}^{N} \frac{x_i^{\,T} C^{-1} x_i}{2})$$

The log-likelihood function of C is

$$\ln\left(L\left(C;\left\{x_i\right\}\right)\right) = \frac{MN}{2}\ln 2\pi - \frac{N}{2}\ln\det\left(C\right) - \frac{1}{2} tr(C^{-1}\sum_{i=1}^{N} x_i x_i^{\,T})$$

Maximum-likelihood estimation is used here, hence,

$$\hat{C} = \arg\max(L'(C;\{x_i\}))$$

$$= \arg\max(-\ln\det\left(C\right) - tr(C^{-1}\frac{1}{N}\sum_{i=1}^{N} x_i x_i^{\,T}))$$

Estimation of the covariance matrix

The matrix $S = \frac{1}{N}\sum_{i=1}^{N} x_i x_i^T$ is of course a semi-positive definite matrix, and is also symmetric.

If we want to acquire the estimation of C, the following optimization problem should be solved:

- **maximize**: $-\ln\det\left(C\right) - tr(C^{-1}\frac{1}{N}\sum_{i=1}^{N} x_i x_i^T)$
- **subject to**: C belongs to the set of semi-positive definite matrices

It is proved that the above optimization problem will have solutions.

Estimation of the covariance matrix

- How to solve the estimation problem

  If the objective function

$$-\ln\det\left(C\right) - tr(C^{-1}S)$$

achieves its maximum, he total differential of this function will be zero, i.e.,

$$d\left(-\ln\det\left(C\right) - tr(C^{-1}S)\right) = 0$$

Estimation of the covariance matrix

Here, we use two facts:

(1) $d\left(\det\left(C\right)\right) = \sum_{i=1}^{M} \frac{\partial \sum_{j=1}^{M} c_{ij} C_{ij}^{*}}{\partial c_{ij}} dc_{ij} = tr(C^{-1} dC)$

(2) $d(tr(C^{(-1)}S)) = -tr(C^{(-1)} dC C^{(-1)} S)$

where C^{*} is the adjoint matrix of C .

Hence,

$$d\left(-\ln\det\left(C\right) - tr(C^{-1}S)\right) = tr(\left(C^{-1}SC^{-1} - C^{-1}\right)dC)$$

Estimation of the covariance matrix

If C is symmetric, we can substitute dC by C, therefore,

$$d\left(-\ln\det\left(C\right) - tr(C^{-1}S)\right) = 0$$

$$\Updownarrow$$

$$tr\left(C^{-1}S\right) = M$$

where M is the dimension of x_i. Here, the estimator of C is $S = \frac{1}{N}\sum_{i=1}^{N} x_i x_i^T$.

Estimation of the covariance matrix

- Estimation of a complex valued covariance matrix

  In previous sections, when the covariance matrix is real valued, the best estimator of it is the average of the sample matrix. However, the covariance of the received signals has the form of

$$C = C_N + GPG^H$$

  where G is the array manifold matrix which is definitely a complex valued matrix.

Estimation of the covariance matrix

The received signal x_i is complex valued, so we define

$$S = \frac{1}{N}\sum_{i=1}^{N} x_i x_i^H$$

where H denotes the conjugate transpose.

Estimation of the covariance matrix

- Solving estimation problem of complex valued covariance matrix

  We can use the result of previous sections to solve the estimation problem of complex valued scenario.

  The key point to this problem is to construct a real valued, positive definite, and symmetric matrix using S .

Estimation of the covariance matrix

Let $S^R = \frac{1}{N}\sum_{i=1}^{N}\begin{bmatrix}\mathrm{Re}(x_i)\\ \mathrm{Im}(x_i)\end{bmatrix}\begin{bmatrix}\mathrm{Re}(x_i)^T & \mathrm{Im}(x_i)^T\end{bmatrix}$

Obviously, S^R is a real valued, positive definite, and symmetric matrix, and it conveys all the information about S .

Estimation of the covariance matrix

Furthermore, let

$$O = \frac{1}{N}\sum_{i=1}^{N} \mathrm{Re}\left(x_i\right)\mathrm{Re}\left(x_i\right)^T$$

$$P = \frac{1}{N}\sum_{i=1}^{N} \mathrm{Im}\left(x_i\right)\mathrm{Im}\left(x_i\right)^T$$

$$Q = \frac{1}{N}\sum_{i=1}^{N} \mathrm{Re}\left(x_i\right)\mathrm{Im}\left(x_i\right)^T$$

then

$$S^R = \begin{bmatrix} O & Q \\ Q^T & P \end{bmatrix}$$

Estimation of the covariance matrix

Denote the estimator acquired from S^R is $\hat{C}^R$.

The relationship between $\hat{C}^R$ and $\hat{C}$ is one-to-one mapping, i.e., .

$$\hat{c}_{ij} = o_{ij} + p_{ij} - jq_{ij} + jq_{ji}$$

Therefore, the estimation problem is solved through an estimation of real valued covariance matrix.

Future Work

This paper discuss on the theoretical parts of determining the arrival angles of sensed events using the covariance matrix estimation. In future, intensive field experiment should be conducted to verify the result of this paper.

Thank you!